

Introduction

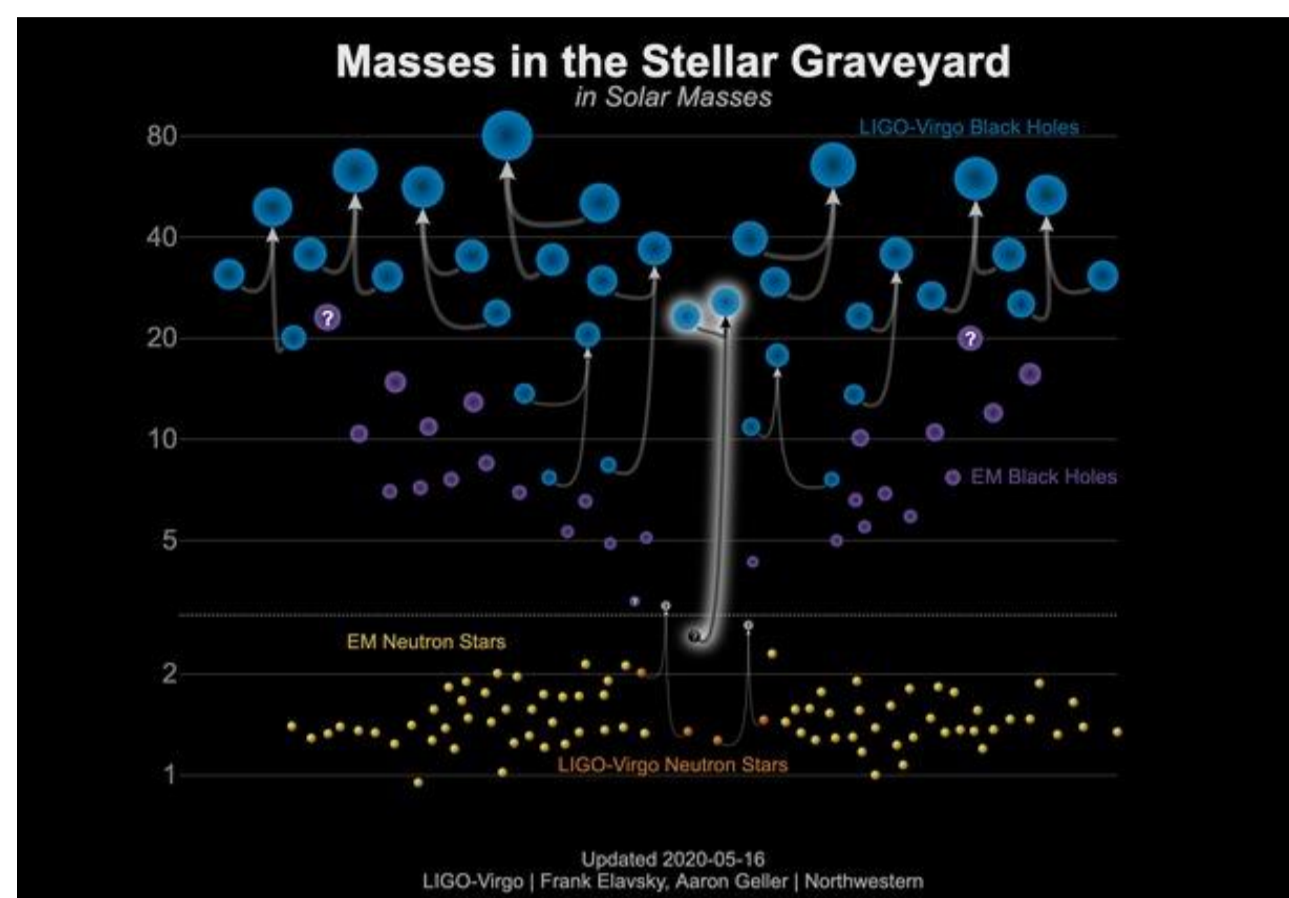
Neutron stars (NS) are exotic state of matter with the highest possible density and compactness in the observable universe, making them valuable laboratories for studying such an extreme state of matter. They are of high interest because they play a crucial role in governing many astrophysical phenomena:

1. The neutron star-neutron star (BNS) and neutron star-black hole merger are the primary sources of gravitational waves.
2. These systems are also thought to be the central engines of short gamma-ray bursts.
3. They are potential sources for electromagnetic and neutrino emissions.
4. Their merger is the source of production for very heavy elements in the Universe
5. They are associated with numerous explosive, transient, and non-electromagnetic events.
6. For testing GR in the strong-field regime.
7. Sources for studying multi-messenger astronomy and gravitational wave cosmology.

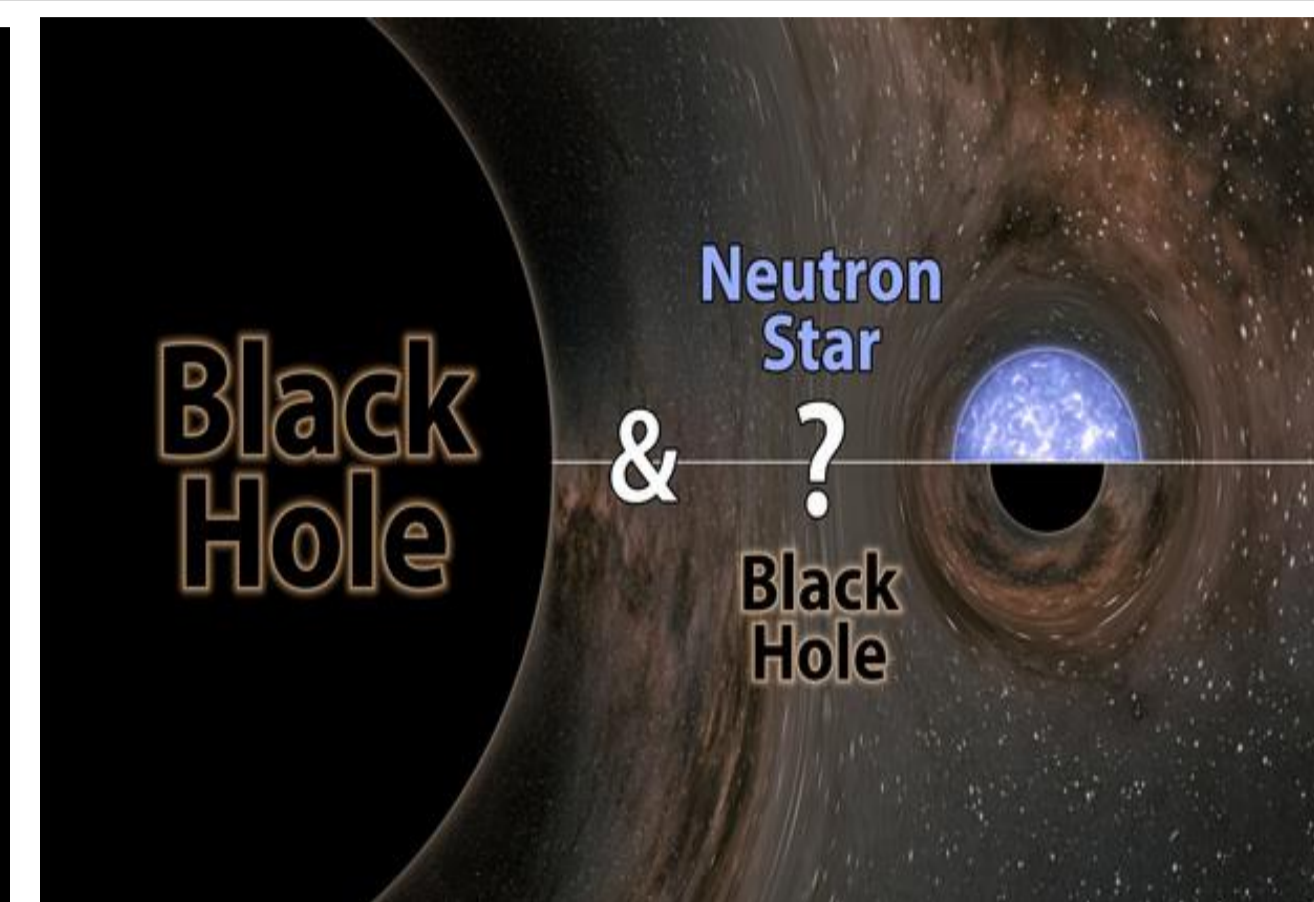
These make them Einstein's richest laboratory in the Universe to explore the relationship between gravity, light, and matter, but the governing equation of state (EOS) for such extreme state is unknown. Yet, we know that one of the most important characteristics of a neutron star is, its maximum allowed mass and even though it is the most important prediction of the general relativistic theory of stellar structures, its limiting value is still an unclear picture from many decades.

Why maximum mass is important?

- Equation of state of NS: Different equations of state allow different maximum masses for neutron stars and, thus, determine the dividing line between neutron stars and black holes. Knowing the maximum possible masses along with the radii can put strong constraints on their EOS made a substantial impact on our understanding of the composition and bulk properties of matter at densities higher than that of the atomic nucleus, a major unsolved problem.
- The 'mass-gap': When the most massive stars die, they collapse under their own gravity and leave behind black holes; when stars that are a bit less massive die, they explode in a supernova and leave behind dense, dead remnants of stars called neutron stars. For decades, astronomers have been puzzled by a gap that lies between neutron stars and black holes: the heaviest known neutron star is no more than 2.5 times the mass of our sun (2.5 solar masses), and the lightest known black hole is about 5 solar masses. The question remained: does anything lie in this so-called mass gap?
- Determining the outcome of binary neutron star merger: Recent detection of gravitational waves (GW190814) is a possible blackhole-neutron star merger. One of the components was an object with the mass of 2.6 solar masses, which is probably too heavy to be a neutron star and is, therefore, more likely to be a black hole. However, we can't rule out the possibility that GW190814 contains an especially heavy neutron star, a scenario that would cause us to dramatically revise our estimates of the maximum possible neutron star mass.

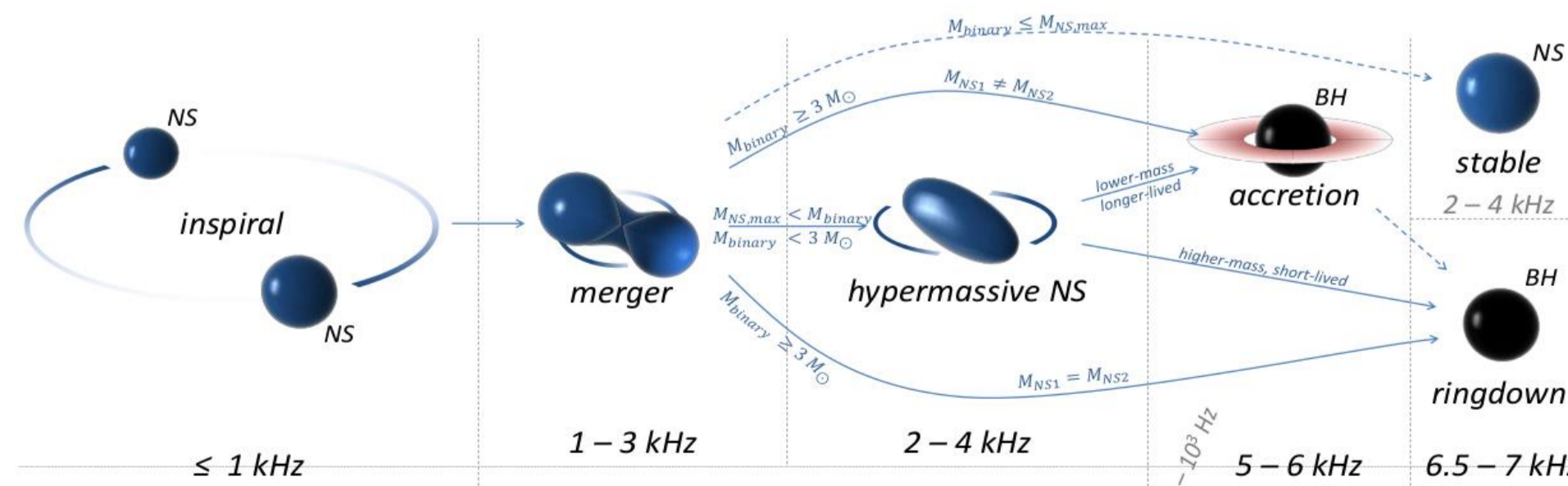


This graphic shows the masses for black holes detected through electromagnetic observations (purple), the black holes measured by gravitational-wave observations (blue), the neutron stars measured with electromagnetic observations (yellow), and the neutron stars detected through gravitational waves (orange). GW190814 is highlighted in the middle of the graphic as the merger of a black hole and a mystery object around 2.6 times the mass of the sun. Image credit: LIGO-Virgo/ Frank Elavsky & Aaron Geller (Northwestern).



The merger of a black hole with 23 times the mass of our sun and a mystery object 2.6 times the mass of the sun. Scientists do not know if the mystery object was a neutron star or black hole, but either way, it set a record as being either the heaviest known neutron star or the lightest known black hole. Image credit: LIGO/Caltech/MIT/R. Hurt (IPAC)

Recent simulations of the BNS merger (by Baiotti et al. 2017) suggest that the outcome of the merger could be a prompt collapse or will have an intermediate meta-stable product which is a highly massive differentially rotating neutron star; before the final collapse to a stable object i.e. a black hole or a neutron star. Hence, estimating the maximum possible mass of this differentially rotating short-lived remnant is consequently an easier way to put constraints on the maximum life span of this intermediate remnant, estimating the mass of the final stable neutron star, constraining EOS, possibly solving the puzzle of the mass gap and ascertaining the expected gravitational wave signals.



Schematic diagram of the evolution of compact binary coalescences. The frequency of the emitted GW is indicated for the different stages. NS-NS inspirals are observable for a few seconds to minutes. The outcome of the merger depends on significantly on the mass of the progenitors, their mass ratio, equation-of-state (EOS). Credits: Bartos et al. 2012

“What is the highest stable density that matter is allowed to achieve before it implodes and collapses inside its own event horizon, never to be seen again?”

Maximum possible mass of NS

- The maximum mass of a cold, nonrotating, spherical neutron star is uniquely determined by the Tolman-Oppenheimer-Volkoff equations and depends only on the cold equation of state. This maximum mass is found to be in the range of 1.8 – 2.3 solar mass (Akmal, Pandharipande and Ravenhall 1998).
- Rigid Rotation (uniform rotation) can further stabilize slightly more massive stars. This maximum mass of rotating neutron star was found to be at most ~ 20% larger than the nonrotating value. For rigidly rotating NSs the maximum allowed mass can be 14% - 22% larger than non-rotating (Cook, Shapiro, & Teukolsky, 1992, 1994a, 1994b).
- Differential rotation can further increase the maximum allowed mass of neutron stars and may temporarily stabilize the remnant of binary neutron star mergers. It is found to be much more efficient to increase the maximum mass (more than ~60%) because the star's core may then rotate faster than the envelope and such a star could support a significantly larger mass than its uniformly rotating or non-rotating counterpart (Baumgarte et al. 2000 [5]; Lyford et al. 2003 [6])

Interesting observations were :

1. The fractional increase of the maximum allowed mass was larger for moderately stiff equations of state (EOS).
2. For each EOS, was not a monotonic function of the degree of differential rotation \tilde{A} : it was first increasing with it, reaching a maximum value for an intermediate \tilde{A} , and then decreasing.

This behaviour was highly non-linear. In contrast to the maximum mass of the nonrotating and uniformly rotating stars, the maximum mass of differentially rotating stars cannot be uniquely defined, since the value will depend on the chosen differential rotation law as well as EOS and \tilde{A} .

The whole solution space of differentially rotating axisymmetric neutron stars was first explored in Ansorg et al. (2009), using a relativistic, highly accurate double-domain pseudo spectral code (based on the so-called ‘AKM-method’; Ansorg, Kleinwächter & Meinel 2003a).

Equation of State and Rotation Law used in AKM method

1. Modelling the star as a stationary and axisymmetric relativistic star in differential rotation

2. Polytropic equation of state : $P = K\rho^\Gamma$

where, P is the pressure, ρ is the rest-mass density, K is the polytropic constant and Γ is the polytropic index ($\Gamma=1.8$ to 3) to study the effects of the stiffness of EOS on the maximum mass.

3. Astrophysically motivated **J-constant law as rotation profile**, (Komatsu et al. 1989) which implies an angular velocity that monotonously decreases from the axis of rotation to the equator;

$$F(\Omega) = A(\Omega_c - \Omega)$$

Where, Ω is the angular velocity of the star, Ω_c is angular velocity at the axis of rotation

Where, $\tilde{A} = r_e/A$: is the Degree of differential rotation described by a dimensionless parameter \tilde{A} ,

$\tilde{A} = 0$ corresponds to the rigid rotation, $\Omega = \Omega_c$ in the whole star;

$\tilde{A} = 1$ corresponds to the angular velocity at the equatorial surface is around $\Omega = \Omega_c / 2$

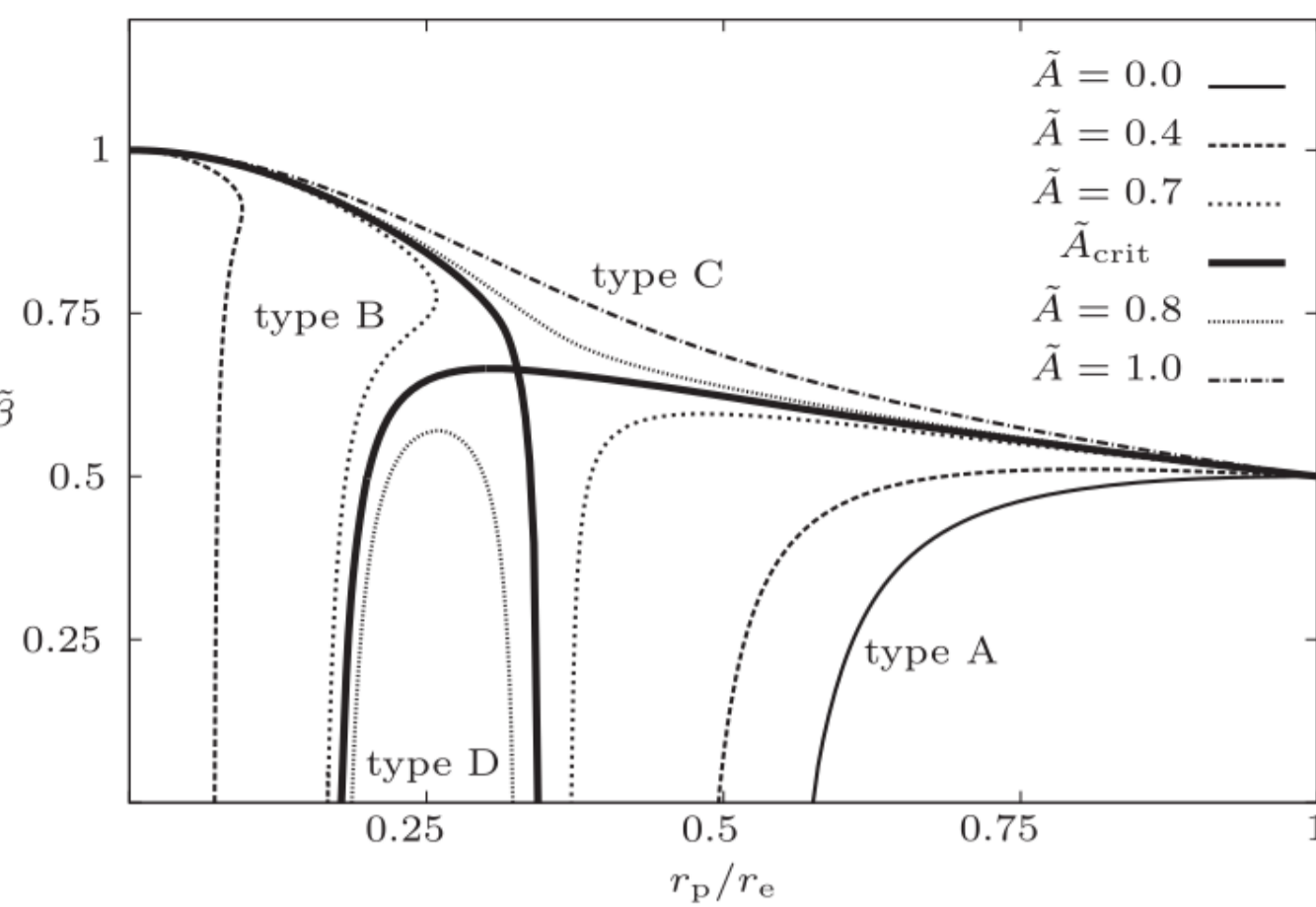
During core collapse of the star(or merger product) the average \tilde{A} is 0.7-0.9.

4. The degree of differential rotation is an increasing and monotonic function of \tilde{A} .

1. A noticeable result was the discovery of four ‘types’ of configurations that co-exist with each other.

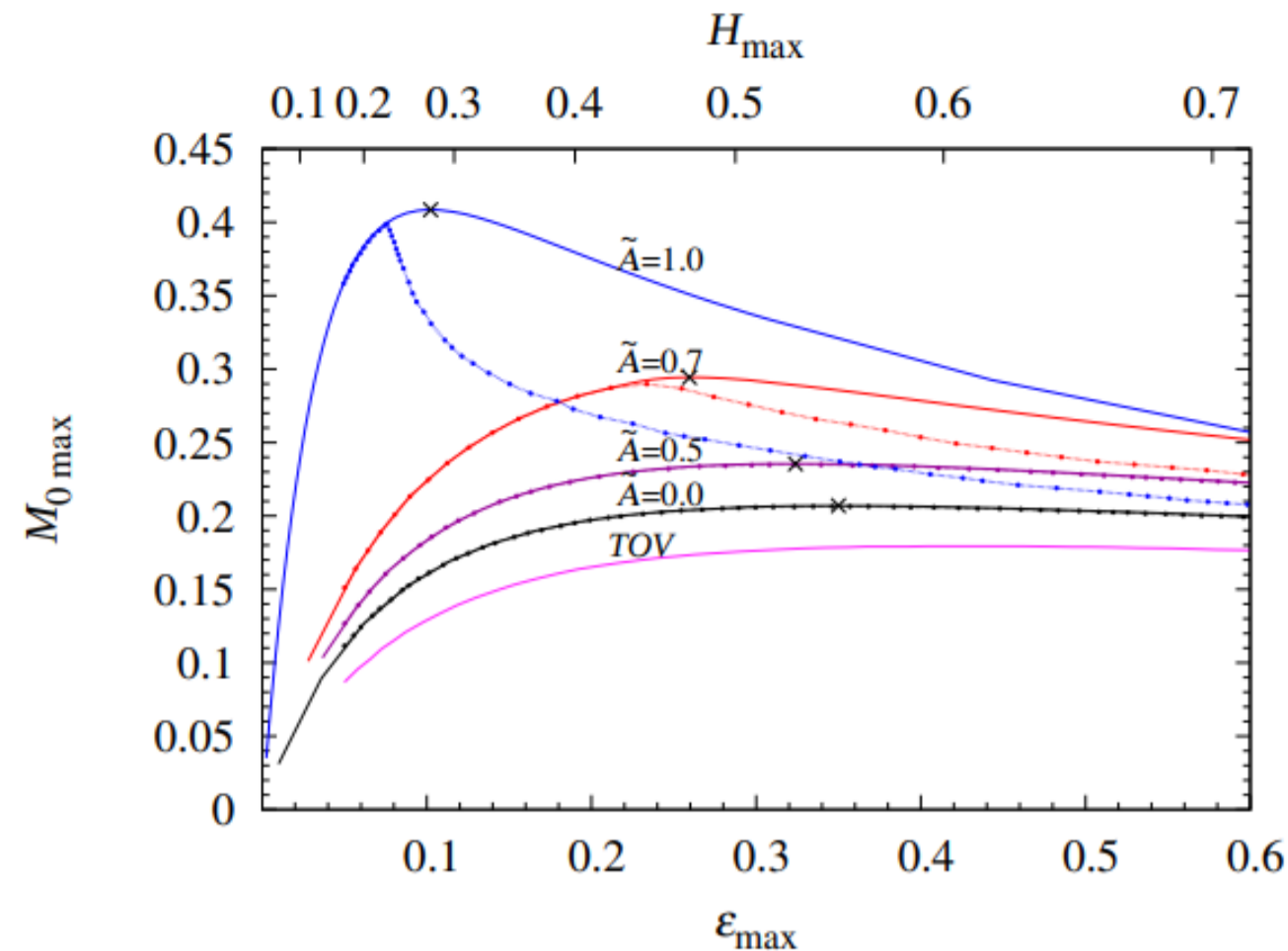
2. The maximum mass and various other astrophysical parameters were calculated for all types of differentially rotating neutron stars modelled by a $\Gamma = 2$ polytropic EOS in Gondek-Rosinska et al. 2017 for the first time. For this particular EOS, the maximum mass of differentially rotating neutron stars was shown to depend not only on the \tilde{A} , but also on the type of the solution. It was shown that the maximum mass is an increasing function of \tilde{A} for type A solutions and a decreasing one for types B, C, and D.

3. This behavior was found also for other polytropic EOS (Studzińska et al. 2016) and recently confirmed by Espino & Paschalidis (2019) and by Szkudlarek et al. 2019 for a realistic EOS and strange quark matter EOS respectively.



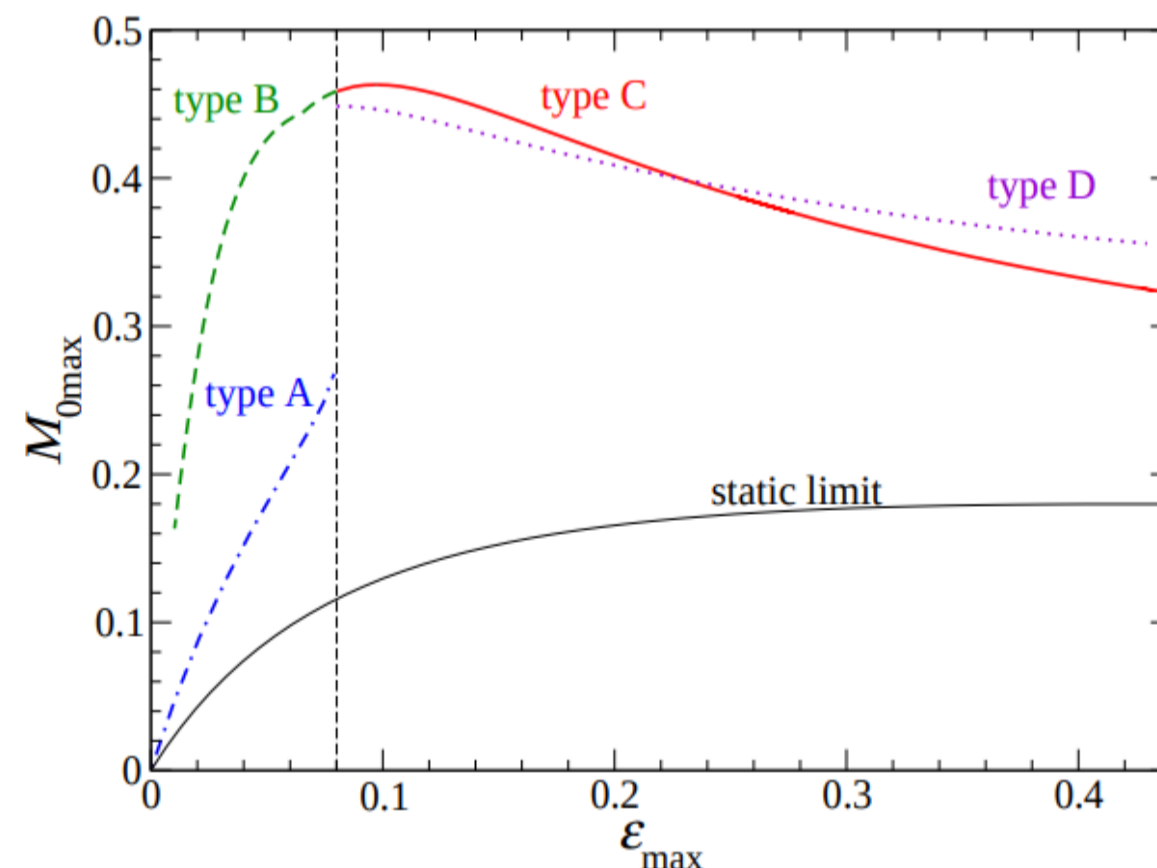
A noticeable result was the discovery of ‘four’ types of solutions, that co-exists! (Ansorg et al. 2009)

- The figure shows the typical structure of the solution space illustrating the various types of sequences for several values of the degree of differential rotation \tilde{A} in β - r _ratio plane. The curves show the dependency between the shedding parameter β and the ratio between polar and equatorial radii r _ratio for $\Gamma = 2$ polytropic stars with fixed maximal energy density $\epsilon_{\max} = 0.12$ (maximal enthalpy $h_{\max} = 0.2$). The thick curve corresponds to the separatrix sequence with $\tilde{A}_{\text{crit}} = 0.75904$, which divides the diagram into four regions containing sequences of types: A (lower-right corner), B (lower-left corner), C (above separatrix), and D (between types A and B).
- The key difference between rigidly and differentially stationary rotating stars is that the solution space for the later is much richer, for all considered EOS .
- Types B, C and D exist only due to differential rotation, these configurations were not obtained in previous studies.
- Type A : exist for $\tilde{A} < \tilde{A}_{\text{cr}}$. These sequences start at a static and spherical body and end at the mass-shedding limit . They exist for all the EOS considered ($\Gamma=1.8$ to 3)
- Type B : exist for $\tilde{A} < \tilde{A}_{\text{cr}}$ and $\tilde{A} > \tilde{A}_{\text{B}}$. They start at the mass-shedding limit and end at r _ratio =0. They exist only for r _ratio < 0.3 and moderately stiff EOS.
- Type C : exist for $\tilde{A} > \tilde{A}_{\text{cr}}$. These sequences start at a static and spherical body and end at the r _ratio =0. for configurations with spherical topology.
- Type D : $\tilde{A} > \tilde{A}_{\text{cr}}$ but only in a narrow range of \tilde{A} . Its two extremities are at the mass-shedding limit.



Differential rotation increased maximum masses from over 3 to over 4 times more compared to non-rotating star! (Gondek-Rosinska et al. 2017)

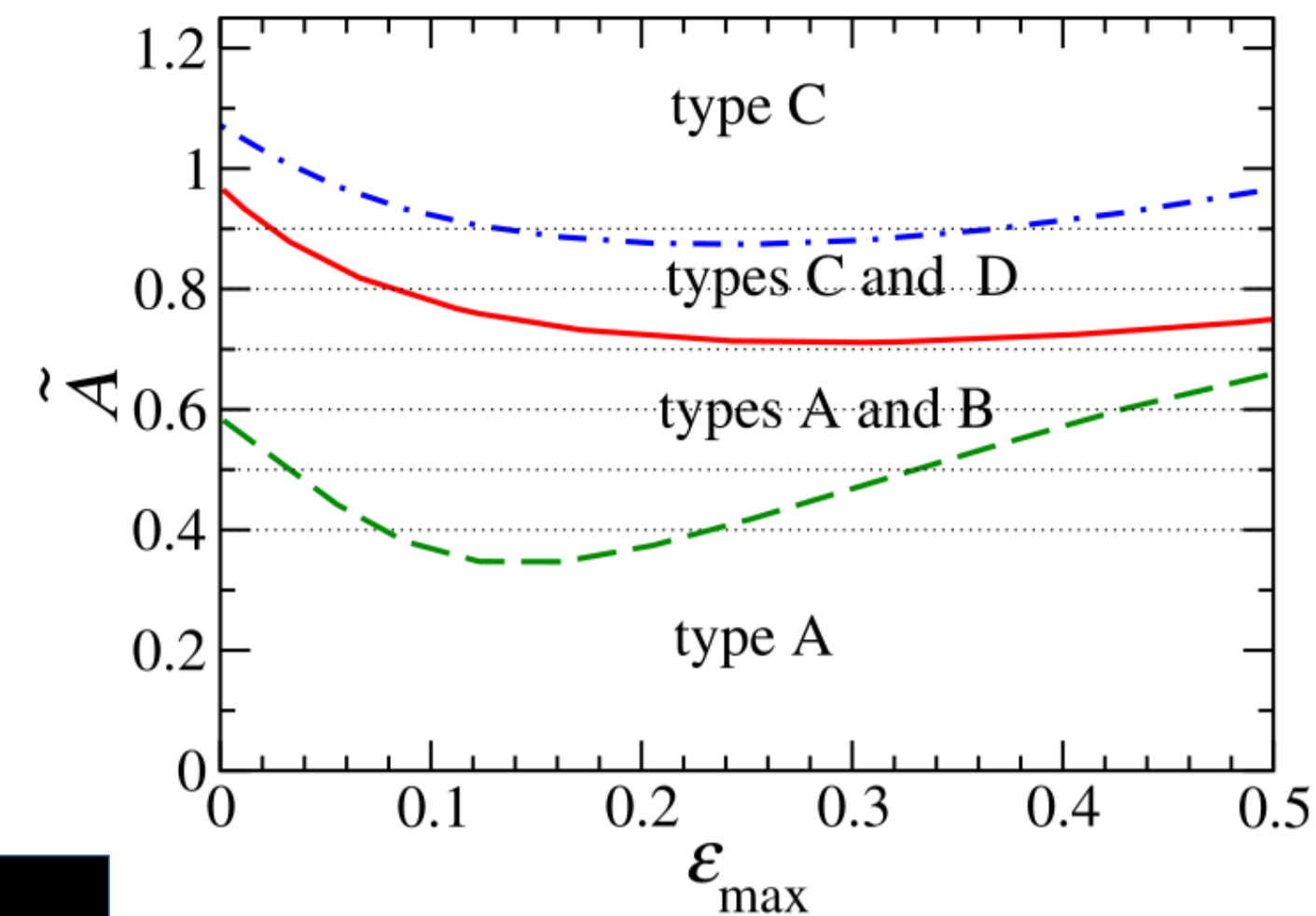
Maximum rest mass as a function of maximal energy density (maximal enthalpy) for NS described by EOS with $\Gamma=2$ with various degrees of differential rotation. The solid black line corresponds to the static (no rotation, TOV) configuration . $\tilde{A}=0$ corresponds to the mass-shedding limit for rigid rotation. We see that the differential rotation increased maximum masses from over 3 to 4 times more compared to non-rotating star. (x) corresponds to the value of maximum mass.



Dependence on Type of Solution

Maximum mass for each Type of solutions for a differentially rotating NS (moderate) with fixed degree of differential rotation ($\tilde{A} = 0.8$) is shown, for the polytropic EOS with $\Gamma=2$.

The maximum mass for Type B, is two times larger than that Type A solutions (however, some of the configurations could be unstable) . (Gondek-Rosinska et al. 2017)



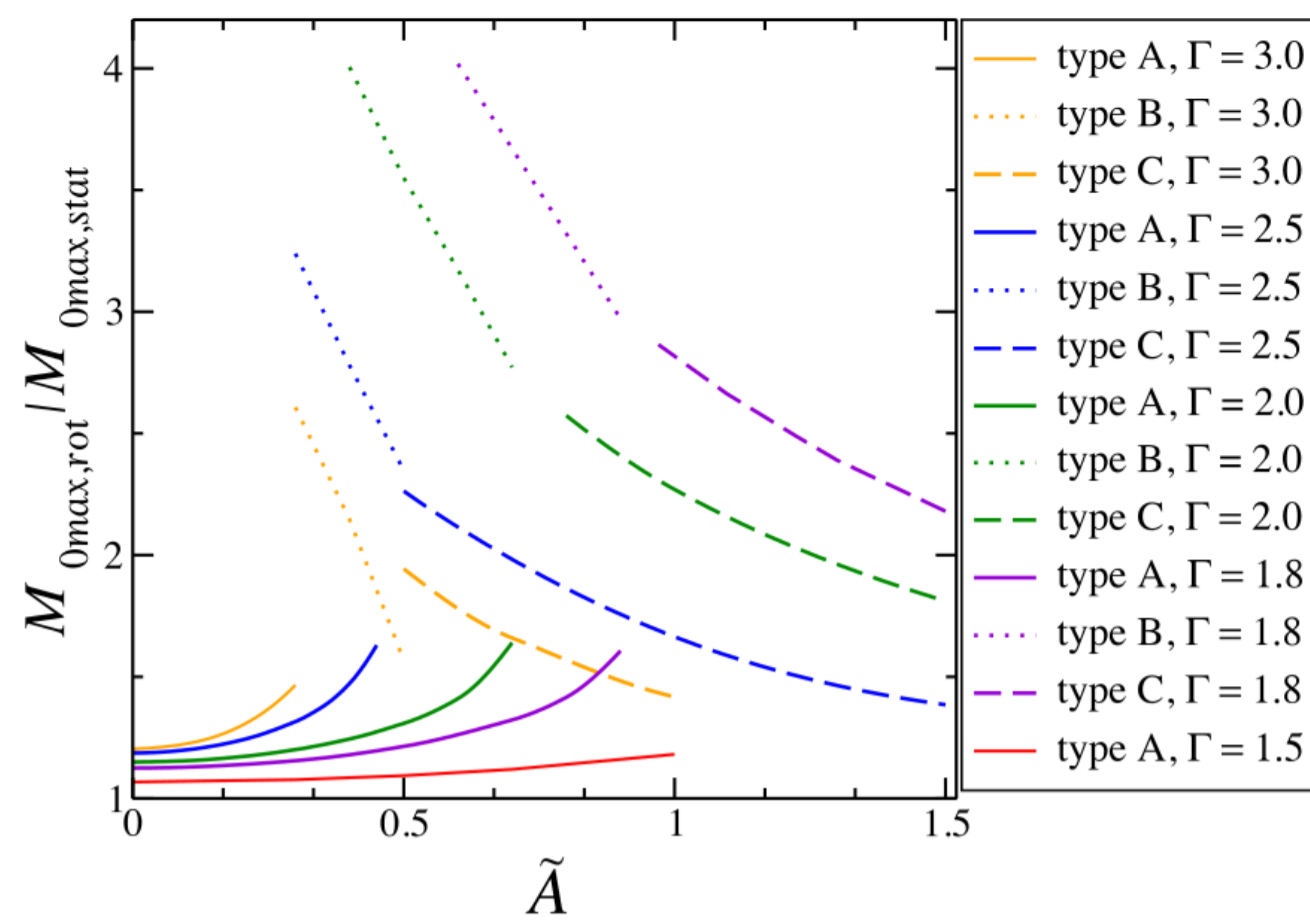
Co-existing solutions!

For small degrees of differential rotation only Type A exist, for higher values both Type A and Type B exist, then types C and D for even higher values, and for sufficiently high degree only Type C exist. All the types of solutions exist for stiff and moderately stiffer EOS. (Gondek-Rosinska et al. 2017)

Figure shows the regions of existence of A, B, C, and D types of differentially rotating NS sequences in the place (\tilde{A} - ϵ_{\max}). The central (red) curve indicates the critical value \tilde{A}_{cr} , which corresponds to the separatrix, on which all types (A, B, C, and D) of solutions coexist. This critical value of degree of differential rotation \tilde{A} (\tilde{A}_{cr}) characteristic to given EoS, which divides solution space into four different types;

1. $0 < \tilde{A} < \tilde{A}_{\text{B}}$ - all sequences are of type A;
2. $\tilde{A}_{\text{B}} < \tilde{A} < \tilde{A}_{\text{cr}}$ - all sequences are of type A or B;
3. $\tilde{A}_{\text{cr}} < \tilde{A} < \tilde{A}_{\text{D}}$ - all sequences are of type C or D;
4. $\tilde{A}_{\text{D}} < \tilde{A}$ - all sequences are of type C.

Note that for value of $\tilde{A} < \tilde{A}_{\text{B}}$ or $\tilde{A} > \tilde{A}_{\text{D}}$ only one type exist, A or C respectively.



The stability of neutron stars is a classical problem in general relativity and one of its most important results is the so-called “turning-point criterion” by Friedman et al. (1988). It states that along a sequence of nonrotating relativistic stars, secular instability sets in at the maximum of this sequence, i.e., at the turning point. On the other hand, Takami et al. (2011) found that for uniformly rotating stars this is just a sufficient, not a necessary criterion. An important issue is the stability of these the question of these massive configurations of differentially rotating neutron stars.

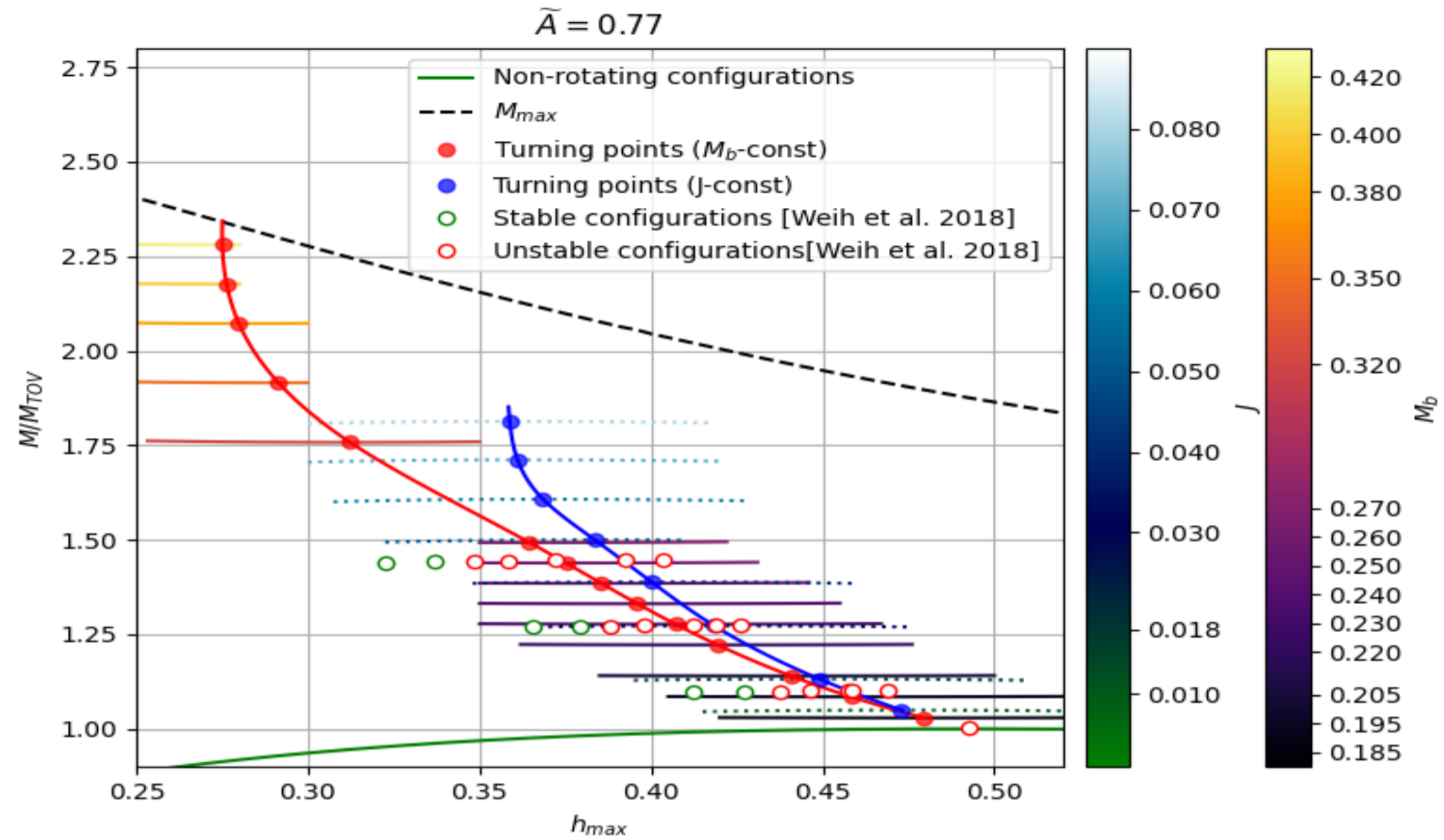
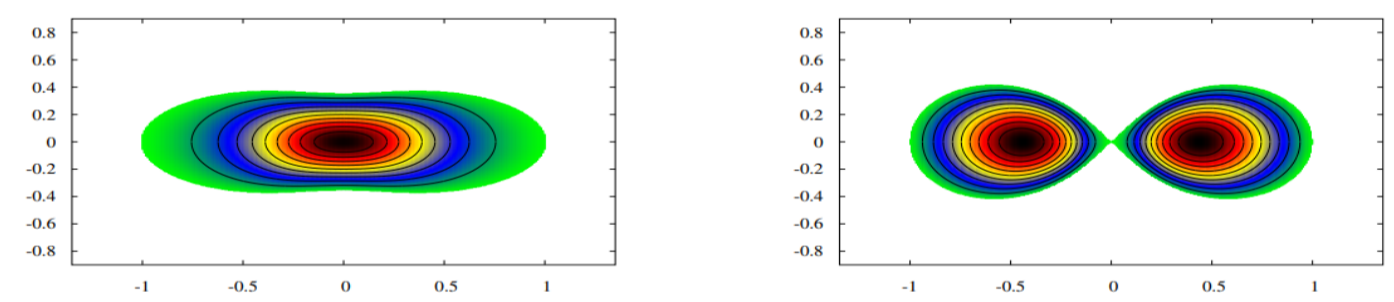


Figure shows the ratio of the maximum allowed rest mass of differentially rotating NS to the maximum mass of non-rotating stars, versus the degree of differential rotation \tilde{A} for EOS with different stiffness ($\Gamma=1.5$ to 3) Studzińska et al. 2016

We see that the largest increase of mass is for moderately stiff EOS, for Type B. The maximum mass for differentially rotating NS could be even 3-4 larger than the static using Komatsu et al law for differential rotation. One should remember that this law is more realistic for new born NS than for differentially rotating remnant of a merger of two NS.

The stiffer the EOS is, the lower \tilde{A} has to be, for the Type B, C, D to exists.



Shape of the stars (meridional cross sections of stars) with the maximum allowed mass. $\tilde{A} = 0.7$ (Type A, left), $\tilde{A} = 1$ (Type C, right). Differentially rotating NSs may be extremely flattened and maximum density may not exist at the center. (Gondek-Rosińska et al. 2017)

The figure above shows Type C solution ($\tilde{A} = 0.77$) maximum mass on the y-axis and maximum enthalpy on the x-axis. Different sequences (curves) on the plot correspond to different values of fixed J (angular momentum) and fixed M (baryon mass). The red solid line is for constant M connecting the maximum mass of each sequence and similarly blue solid line is for constant J. Green curve corresponds to non-rotating star and black-dashed line corresponds to maximum mass of differentially rotating star ($\tilde{A}=0.77$). The red circles are unstable configurations that collapse to BH and green ones are stable ones, taken from (Weih et al. 2018). As it can be seen that the red line (M const) is close to marginally stable configurations found by Weih et al. (using turning-point criterion) indicate that massive differentially rotating NS could indeed be stable and the maximum mass could be in the stable Region. **Credit : P.Szewczyk, D.Gondek-Rosińska, P.Mehta (In preparation)**

Due to the nonlinear nature of this system of equations, there is nevertheless not always uniqueness of the solution if those are indeed the fixed quantities, a feature that makes the solution space even richer than it is for rigidly rotating star

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