Linear vs. nonlinear methods for detecting magnetospheric and ionospheric current systems patterns

T. Alberti[1], F. Giannattasio[2], P. De Michelis[2], G. Consolini[1]

[1]INAF-IAPS, Via del Fosso del Cavaliere 100, 00133, Roma, Italy

[2]Istituto Nazionale di Geofisica e Vulcanologia, via di Vigna Murata 605, 00143, Roma, Italy

Abstract

We apply both the EOF and the MEMD methods to characterize spatial patterns of external vertical component of the geomagnetic field at low and middle latitudes as a function of geomagnetic activity level recorded by Swarm constellation during a period of two years from 1 April 2014 to 31 March 2016. We show how MEMD method is particularly suitable to avoid misleading results produced by the EOF one. Indeed, the MEMD method extracts the main modes which contribute to the determination of the geomagnetic field of external origin during quiet periods by using few modes to recognize the different contributions coming from sources external to the Earth in the magnetic signal and to reproduce the realistic equivalent currents responsible for the recorded magnetic field variations. This study is an example of the potential of MEMD method which can be used to give new insights into the analysis of the different sources responsible of the geomagnetic field of external origin and, at the same time, it can be used as a good filter in the analysis of the geomagnetic field of external origin permitting us to separate the ionospheric contribution from the magnetospheric one [1,2].

Swarm data

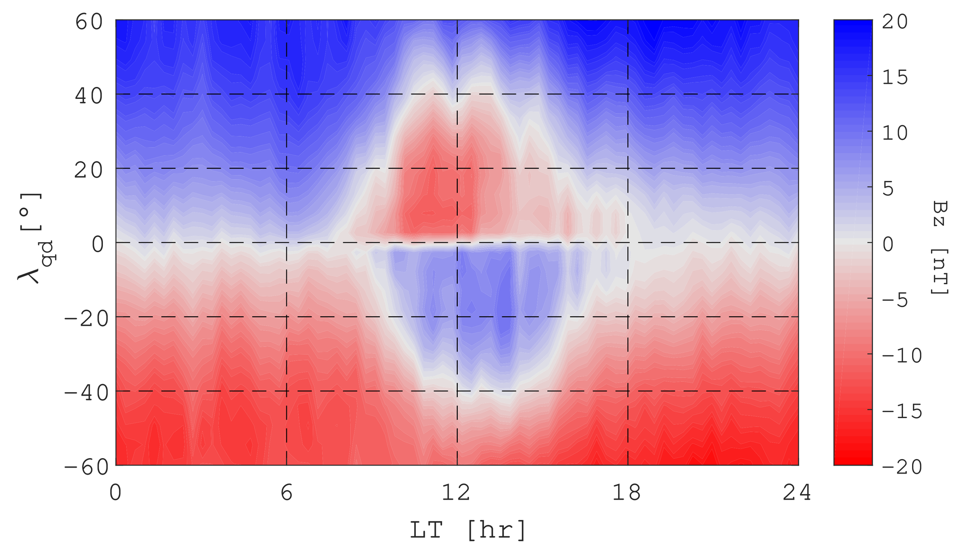

Figure 1: Global map of the geomagnetic field of external origin along the vertical component in the λ_{qd}–LT plane.

- Level-1b low resolution (1 Hz) vector magnetic field data recorded on Swarm A satellite during a period of two years from 1 April 2014 to 31 March 2016 (`ftp://swarm-diss.eo.esa.int`) [3]
- we analyzed the vertical component of the geomagnetic field (B_z) at low- and mid-latitudes (within $\pm$ 60° magnetic latitude) recorded during periods characterized by very low geomagnetic activity levels with $AE < 80$ nT and $-10\text{nT} < SYM - H < 5\text{nT}$
- we removed the internal geomagnetic field from the original data by using CHAOS-6 model [4]
- external vertical field ranges between -20 and 20 nT and a two-lobe structure is clearly visible
- it is consistent with the solar quiet (S_q) daily variation of the geomagnetic field, a regular variation due to electric currents flowing in the ionosphere
- it generates an induced magnetic field along $\hat{\mathbf{z}}$ directed outward in the Northern Hemisphere and inward in the Southern Hemisphere, in both cases opposite to the main geomagnetic field vertical component, and thus it is revealed by Swarm observations as a decrease of the geomagnetic field in the $\hat{\mathbf{z}}$ direction in the Northern Hemisphere and an increase in the Southern Hemisphere

Empirical Orthogonal Function (EOF) analysis

- a decomposition technique for both univariate and multivariate data [5]
- the set of observations $\{\mathbf{s}(t)\}|_{t\in T} = \{s_1(t), s_2(t), \ldots, s_n(t)\}$ is converted into a set of values of linearly uncorrelated variables, i.e., the PCs $\Phi_l(t)$, as

$$\{\mathbf{s}(t)\}|_{t\in T} = \sum_{l=1}^{n} \Phi_l(t)\mathbf{L}_l^{\mathrm{T}} \qquad (1)$$

- $\mathbf{L}_l^{\mathrm{T}}$ the transpose of the $l-$th eigenvector of the covariance matrix of $\{\mathbf{s}(t)\}|_{t\in T}$ obtained as $\mathbf{C} = \{\mathbf{s}\}^{\mathrm{T}}\{\mathbf{s}\}$
- the decomposition is complete and orthogonal (by construction)
- the normalized eigenvalue ϵ_l captures the partial variance (i.e., the energy content) of the $l-$th principal component
- summaryzing, the main steps of the EOF method are:
  1. to organize data as a matrix (by using the embedding theorem for univariate data);
  2. to evaluate the covariance matrix of data (embedded data for univariate data);
  3. to diagonalize the covariance matrix to find eigenvectors and eigenvalues;
  4. to project data on eigenvector directions to find the uncorrelated variables, i.e., the principal components.

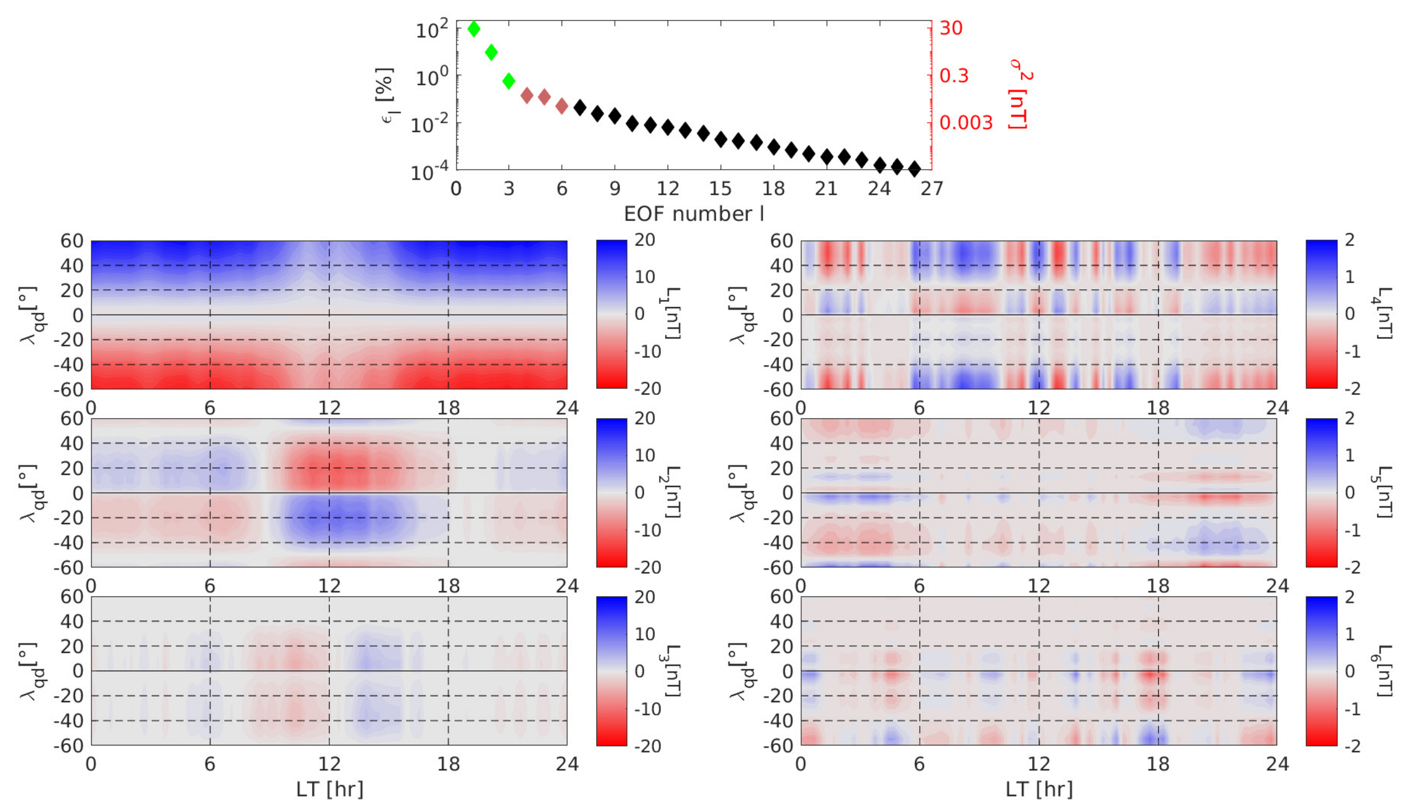

Figure 2: Empirical Orthogonal Function analysis of Swarm data. (on the top) Percentage contribution of EOFs and their variance. To the left, the first three EOFs corresponding with green diamonds in the top panel, and to the right EOFs 4-6 corresponding with orange diamonds in the top panel.

- the first three components (from L_1 to L_3) are characterized by large scale spatial patterns
- the most energetic contribution given by L_1 does not reproduce the main spatial pattern that is visible in the original data associated to the S_q daily variation
- L_1 is characterized by a symmetric spatial pattern both in latitude and in LT
- L_2 is characterized by a two-vortex like structure centered around noon and symmetric with respect to the geomagnetic equator, in agreement with the S_q main pattern structure
- L_3 seems to be characterized by a symmetric pattern in λ_{qd}, with no evidence of LT symmetry
- $L_4 - L_6$ show striped patterns, characterized by latitudinal ribbons of alternate positive and negative amplitudes, while the remaining components (not shown) can be attributed to the noise, due to the low variance

Multivariate Empirical Mode Decomposition (MEMD)

- it works directly in the data domain rather than in an associate one [6]
- the set of observations $\{\mathbf{s}(t)\}|_{t\in T} = \{s_1(t), s_2(t), \ldots, s_n(t)\}$ is decomposed into a set of nonlinear multivariate empirical modes $\{\mathbf{c}_k(t)\}|_{t\in T}$ as

$$\{\mathbf{s}(t)\}|_{t\in T} = \sum_{k=1}^{N} \{\mathbf{c}_k(t)\}|_{t\in T} + \{\mathbf{r}(t)\}|_{t\in T} \qquad (2)$$

- the set of n-dimensional embedded patterns $\{\mathbf{c}_k(t)\}|_{t\in T}^{i=1,\ldots,N}$ empirically forms decomposition basis and $\{\mathbf{r}(t)\}|_{t\in T}$ is the decomposition residue
- MEMD modes empirically and locally satisfy orthogonal and completeness properties
- the energy content e_k of each MEMD mode can be evaluated as $\langle\{\mathbf{c}_k(t)\}|_{t\in T}, \{\mathbf{c}_k(t)\}|_{t\in T}\rangle$ and capture the partial variance
- summaryzing, the main steps of the MEMD method are:
  1. to identify the local extrema (corresponding to data points where abrupt changes are observed);
  2. to separately interpolate both local maxima and minima by using a cubic spline;
  3. to obtain the mean envelope e_m from maxima and minima interpolations;
  4. to evaluate an Intrinsic Mode Function.

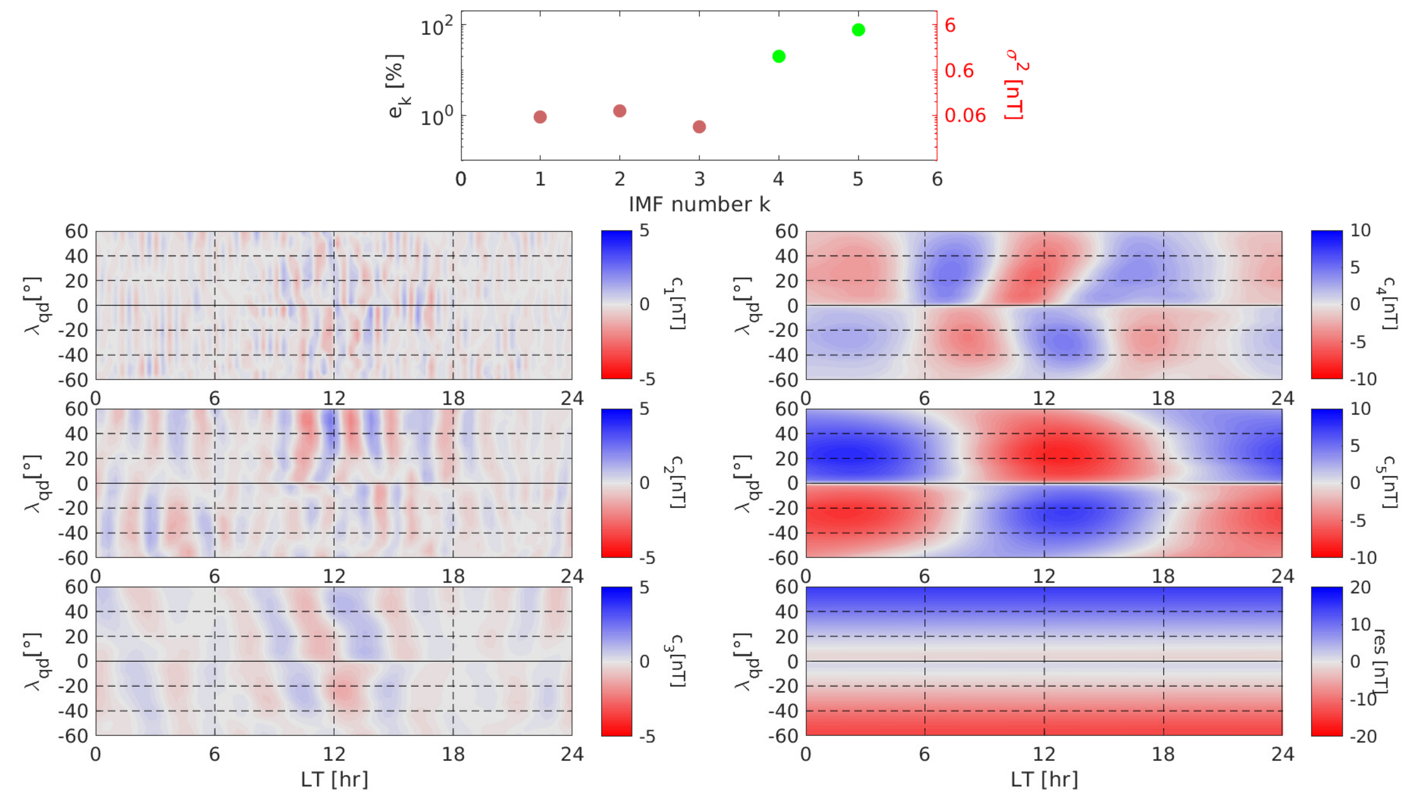

Figure 4: Multivariate Empirical Mode Decomposition analysis of Swarm data. Relative contribution of MEMD modes and their variance (top panel), first three MEMD modes (c_1-c_3, left panels) corresponding with orange diamonds in the top panel, while MEMD modes c_4 and c_5 are reported in the right panels (green diamonds in the top panel).

- the number of detected IMFs and their characteristic spatial frequency are automatically found by the algorithm, being the procedure completely adaptive
- c_1, c_2 and c_3 are characterized by an amplitude in the range $\pm$5 nT and their spatial structures are similar to latitudinal ribbons characterized by alternating positive and negative amplitudes
- the large scale patterns c_4 and c_5 in the maps have strengths spanning the range from $\sim\pm$5 to $\sim\pm$10 nT and features which are the pattern decomposition of the main structure originated by the S_q current in quietness, being c_5 the main component and c_4 its harmonics
- the residual of the original map ranges between $\sim\pm$20 nT, is positive in the Northern Hemisphere and negative in the Southern one, implying that it is inward in the Northern Hemisphere and outward in the Southern Hemisphere $\Rightarrow$ magnetospheric ring current

Comparison between EOF and MEMD results for detecting the S_q variability

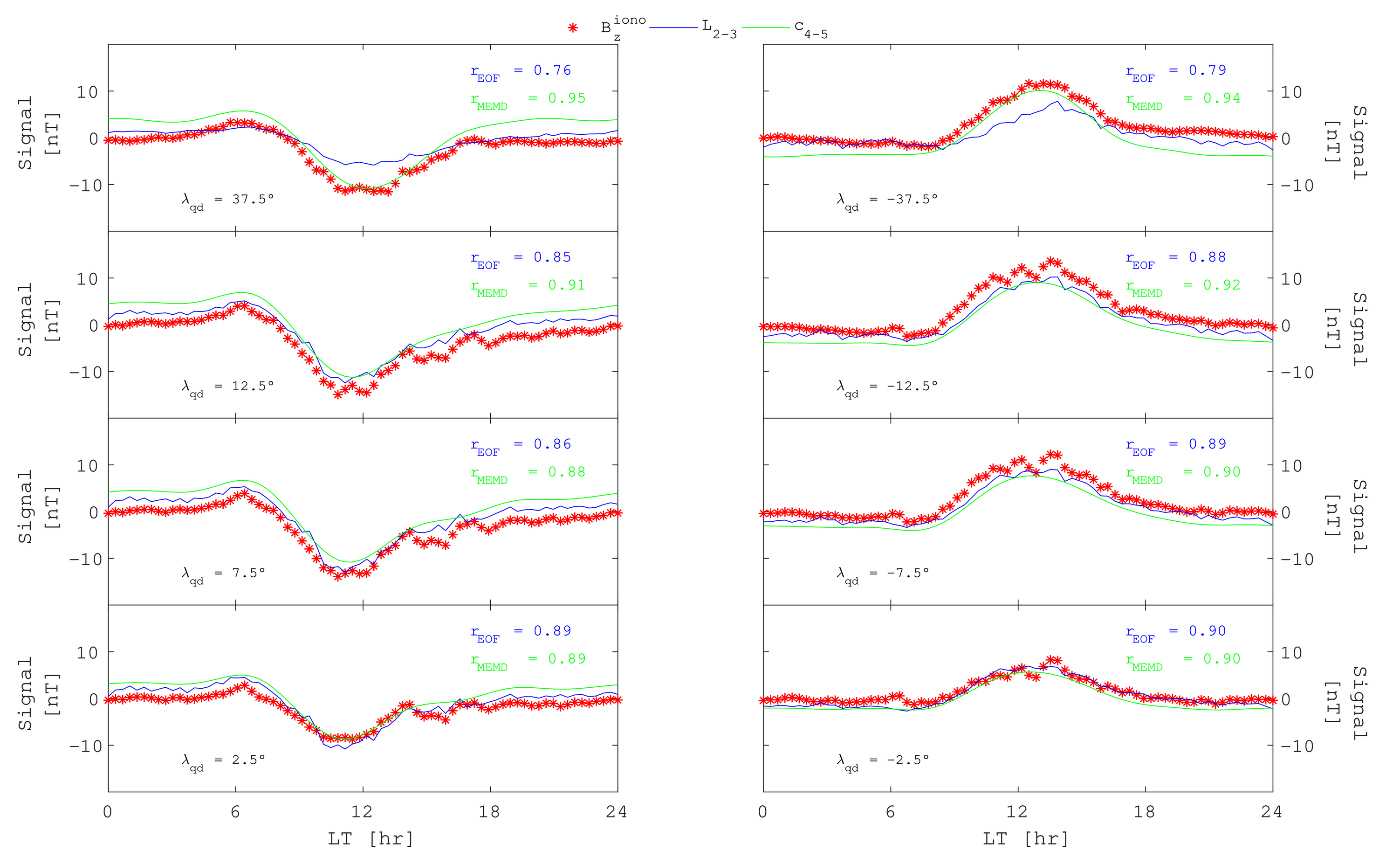

Figure 3: Original data without the magnetospheric contribution (B_z^{iono}, red asterisks), summed EOFs ($L_2 + L_3$, blue line), summed IMFs ($c_4 + c_5$, green line) as a function of the local time LT at different QD latitudes. r_{EOF} and r_{MEMD} refer to the values of correlation coefficient between B_z^{iono} and S_q reconstructions by using EOF (blue text) and MEMD (green text).

Conclusions

- it seems that MEMD method can help in the interpretation of the external magnetic field signals better than EOF method
- the first three modes, i.e., those characterized by the highest spatial frequencies in LT, appear in form of spurious North-South patterns
- c_4 and c_5, i.e., the modes with the lowest spatial frequencies, describe the effects on the geomagnetic field of the electric currents flowing in the ionosphere, i.e., mainly the S_q ionospheric current pattern (see Figure 3)
- the residual, which represents the long-term trend of the analyzed time series, is due to the electric currents flowing in the magnetosphere and describes the effect of the magnetospheric ring current

References

[1] Alberti, T., *Il Nuovo Cimento*, **41 C**, 113, 2018.
[2] Alberti, T. et al., *Earth and Space Science*, **7**, e2019EA000559, 2020.
[3] Friis-Christensen, E., et al., *Earth, Planets and Space*, **58**, 351, 2006.
[4] Finlay, C. C., et al., *Space Sci. Rev.*, doi: 10.1007/s11214-016-0285-9, 2016.
[5] Ghil, M., et al., *Rev. Geophys.*, **40**, 1, 2002.
[6] Rehman, N., and Mandic, D. P., *Proceedings of the Royal Society A*, **466**, 1291, 2010.

Acknowledgments: These results rely on data collected by one of the three satellites of the Swarm constellation which are freely available at `ftp://swarm-diss.eo.esa.int` upon registration. We thank the European Space Agency that supports the Swarm mission.

E-mail contacts: tommaso.alberti@inaf.it